Tips for solution writeups

• Use enough words and sentences. *Every math expression should be part of a sentence with words in it.*
  
  – If an equation is on its own, I don’t know whether you’re saying it’s something that you know to be true, something you are claiming to be true, something you want to prove, or something you want to disprove.

• Be sure that the *idea* behind the proof is clear. If the proof only works because the number you are multiplying by is positive, you should point out that it is positive.

• You should be able to justify every step with a concrete explanation. Explanations like, “because that’s how multiplication works” are not specific enough. Instead, say something like, “because multiplication by a nonnegative number preserves the direction of inequalities.” This can also help you catch errors (like when you forget about the nonnegative condition).
  
  – Also, your justification should not just be a restatement of the thing you are trying to justify if it isn’t something that we already know to be true. You cannot justify a statement like $x^2 \geq x$ by saying “squaring a number makes it bigger.”

• Use precise language. Saying something like “products grow faster than sums” is not very precise. Instead, you should write down the specific inequality you need using actual variables or numbers.

• Quantifiers:

  – If you are trying to *prove* a statement of the form “For all $x \ldots$,” or if you are trying to *disprove* a statement of the form “There exists $x$ such that $\ldots$,” then *examples do not constitute a proof.* You must provide a general argument, so *you need not include any specific examples.* Of course, working out examples can still be helpful when thinking about a problem, just not for the writeup.

  – If you are trying to *disprove* a statement of the form “For all $x \ldots$,” or if you are trying to *prove* a statement of the form “There exists $x$ such that $\ldots$,” then *a single example is sufficient.* You do not need to include how you came up with this example, but you should make sure to do the calculation to show that this example works.

• Please make sure your writeups are legible and organized.

• Recognize when you need to prove an *implication* versus an *equivalence.* Remember that an implication is weaker than an equivalence.

  – We say that $P$ *implies* $Q$ if $Q$ is true whenever $P$ is true; it does not matter whether $Q$ is true or false if $P$ is false.

  – We say that $P$ *and* $Q$ *are equivalent* if $Q$ is true whenever $P$ is true and *vice versa.* Therefore $Q$ is also false when $P$ is false.
• Unless a problem is specifically about logical expressions, operators, etc., you should not use logical notation such as ∧ or ∨ in your proofs.

  – Note also that the symbol ∨ is not interchangeable with the word or in any English sentence. It is only appropriate to use ∨ to connect two logical statements.

    For instance, you might say in English that the minimum of two real numbers \( x \) and \( y \) is \( x \) or \( y \). However, it does not make sense to write \( \min(x, y) = x \lor y \) since \( x \) and \( y \) are real numbers and therefore cannot be combined using \( \lor \). (One could phrase this as the statement \( (\min(x, y) = x) \lor (\min(x, y) = y) \), but it is usually easier just to use an English sentence.)

• Be sure you are using symbols appropriately: arithmetic operations such as + and − can only be used with numbers; set operations such as ∩, ∪, and \( \setminus \) can only be used with sets; logical operations such as ∧, ∨, and \( \setminus \) can only be used with sets, and so forth.

  – Every mathematical object (noun) is of a certain type, such as a number, a set, or a statement. Every mathematical operator (verb) can turn certain types of objects into new objects.

    For instance, the symbol \( \in \) can take a number \( x \) and a set \( S \) to produce the statement \( x \in S \). You cannot write something like \( x \in P \) if \( P \) is a statement. Similarly if \( A \) and \( B \) are sets, you can’t write things like \( \neg A \) or \( A \land B \).