MA 225 Homework 3 – due 9/12

1. Define a logical operator $\bar{\land}$ by declaring that $P \bar{\land} Q$ is true exactly when at least one of $P$ and $Q$ is false.

(a) Suppose $P$ is the statement, “Bob is wearing a T-shirt,” and $Q$ is the statement, “Bob is wearing shorts.” Translate $P \bar{\land} Q$ into a sensible English sentence.

(b) Give the truth table for $P \bar{\land} Q$. Find an equivalent expression for $P \bar{\land} Q$ (using some or all of $\neg$, $\land$, and $\lor$).

(c) Does $(P \land \neg Q) \lor (\neg P \land Q)$ imply $P \bar{\land} Q$?

(d) Simplify each of the following statements into a short, equivalent statement:
   i. $P \bar{\land} P$.
   ii. $(P \bar{\land} Q) \bar{\land} (P \bar{\land} Q)$.

(e) Find a logical expression whose only connective is $\bar{\land}$ that is equivalent to $P \lor Q$. Justify your answer using the equivalence laws discussed in class (DeMorgan’s Laws, idempotency, double negation, and so on).

2. For any two real numbers $x$ and $y$, let $\min(x, y)$ denote the minimum of $x$ and $y$, and let $\max(x, y)$ denote the maximum of $x$ and $y$. For example, $\min(5, 3) = 3$, and $\max(5, 3) = 5$.

Prove that $\min$ distributes over $\max$, that is, for all real numbers $x$, $y$, and $z$:

$$\min(x, \max(y, z)) = \max(\min(x, y), \min(x, z)).$$

(In fact, $\min$ and $\max$ satisfy many of the same properties as $\land$ and $\lor$.)