Let $A$ be a set, and let $R \subseteq A \times A$ be a relation on $A$.
Here are the definitions of some properties that $R$ sometimes satisfies:

- **reflexivity**: for all $x \in A$, $(x, x) \in R$.
- **symmetry**: for all $x, y \in A$, if $(x, y) \in R$, then $(y, x) \in R$.
- **anti-symmetry**: for all $x, y \in A$, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.
- **transitivity**: for all $x, y, z \in A$, if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$.

We say that $R$ is an **equivalence relation** if it satisfies reflexivity, symmetry, and transitivity. We say that $R$ is a **partial order** if it satisfies reflexivity, anti-symmetry, and transitivity.

Suppose $A = \{1, 2, 3, \ldots, 100\}$. Which of the following relations are equivalence relations? Partial orders?

1. $\{(x, y) \in A \times A \mid x \leq y\}$.
2. $\{(x, y) \in A \times A \mid x < y\}$.
3. $\{(x, y) \in A \times A \mid y - x \text{ is even}\}$.
4. $\{(x, y) \in A \times A \mid y - x \text{ is odd}\}$.
5. $\{(x, y) \in A \times A \mid y \text{ is a multiple of } x\}$.
6. $\{(x, y) \in A \times A \mid y \text{ is a multiple of } x \text{ or } x \text{ is a multiple of } y\}$.
7. $\{(x, y) \in A \times A \mid \frac{y}{x} = 2^k \text{ for some integer } k\}$.

Which relations are both an equivalence relation and a partial order? Formulate a theorem and prove it.