Let $P$ be the statement:

“If $x = 5$, then $f(x) = 17$.”

Let $Q$ be the \textit{converse} statement:

“If $f(x) = 17$, then $x = 5$.”

(Assume that we are working in the universe of the real numbers.)

1. (a) Suppose $f(x) = x^2 - 8$. Is $P$ always true? Is $Q$ always true?

(b) Suppose $f(x) = 8 - x^2$. Is $P$ always true? Is $Q$ always true?

2. For each statement below, determine whether it is equivalent to $P$ or $Q$.

(a) $x = 5$ implies $f(x) = 17$.
(b) $x = 5$ if $f(x) = 17$.
(c) $x = 5$ only if $f(x) = 17$.
(d) Only if $x = 5$ does $f(x) = 17$.
(e) $x = 5$ is a necessary condition for $f(x) = 17$.
(f) $x = 5$ is a sufficient condition for $f(x) = 17$.
(g) If $x \neq 5$, then $f(x) \neq 17$.
(h) Either $x = 5$ or $f(x) \neq 17$ (or both).
(i) It is not possible that $x = 5$ and $f(x) \neq 17$.
(j) $x \neq 5$ unless (possibly) $f(x) = 17$. 