1. Associate each letter of the alphabet with an element of $\mathbb{Z}_{26}$ in the following way:

   \[ A = 1, \quad B = 2, \quad C = 3, \quad \ldots, \quad Y = 25, \quad Z = 26 \ (= 0). \]

   For each of the following ciphertexts, find the encryption function $f$, the decryption function $f^{-1}$, and the plaintext given that:

   (a) $f$ is a Caesar shift.

   \[
   \text{WVBPMWBPMPZPIVLGWCIDMLQNNMzMVBNNQVOMZA}
   \]

   (b) $f$ is an affine shift, and the two most common letters in the plaintext are I and T.

   \[
   \text{ZRPDPZCFQYZLHVQZCDABQZWNCQVCNXOQXNQFONXZQ}
   \]

   (c) $f$ is a $2 \times 2$ Hill cipher, and the plaintext ends TQQQ (the Q’s serve as padding).

   \[
   \text{HUHOYZIWNCTRKEU}
   \]

2. Find the greatest common divisor $g$ of $a = 10117$ and $b = 5293$, and find integers $u$ and $v$ such that $g = au + bv$.

3. Let $a$ and $b$ be positive integers. Show that an integer $c$ can be written in the form $c = ax + by$ for some integers $x$ and $y$ if and only if $c$ is a multiple of $g = \gcd(a, b)$. 