1. Find the remainder when $2018^{2017^{2016}}$ is divided by 11.

2. Given that 308911 has prime factorization $541 \cdot 571$, find all four values of $x \in \mathbb{Z}_{308911}$ satisfying the equation $x^2 \equiv 1 \pmod{308911}$.
   (Hint: Solve the equation modulo 541 and 571 separately and use the Chinese remainder theorem.)

3. Given that 2 is a primitive root modulo the prime 1019, is 4 also a primitive root? Is 8? Explain.

4. For this problem, you may find it helpful to use a calculator (or computer algebra system such as MATLAB or Mathematica). Also note that Google can perform many calculations such as $12345 \times 6789 \mod 13579$ typed directly into the search bar.

   Alice wins a trip to meet a certain celebrity, and she wants to tell Bob the good news privately. Since Bob’s public RSA key is $(N, e) = (308911, 175697)$, Alice uses it to encrypt the zip code where she is going, resulting in the encrypted message 66065.

   (a) What is Bob’s private key $d$?
      (Check: $d$ should be a Fibonacci number.)

   (b) What is Alice’s decrypted message? Who is she meeting?