MA 493 Homework 1 – due 8/29

Homework is due at the beginning of class on the date indicated. Collaboration is encouraged, but you are expected to write up your solutions independently. **Use of outside resources is not allowed.** Please indicate with whom you work on each assignment, and be sure to justify all your answers.

If you have any questions about proof-writing or homework problems in general, feel free to send me an email or come see me during office hours.

**Homework:**

1. (a) Alice and Bob take turns (with Alice going first) placing X’s and O’s on a 100×100 board in the usual way. A player wins if they get five of their symbol in a row either horizontally or vertically (not diagonally); if no one wins by the time all the squares are filled, the game ends in a tie. Show that the game will end in a tie assuming best play.

(b) Tired of the previous game, Alice and Bob change the rules so that instead of getting five in a row, the winning player must get six of their symbols into the following shape (rotations and reflections allowed).

![Shape](image)

Show that Alice can either win or tie.

2. Two players take turns choosing different words from the following set:

   \{dime, hips, hold, math, nova, numb, sale, stun, vote\}

A player wins if she manages to collect three words that all share a single letter. If no one wins by the time all the words have been chosen, the game ends in a tie.

What will be the result assuming optimal play? Explain.

3. A knight is placed on the corner of a standard 8×8 chessboard. Two players take turns moving the knight around the board (using standard knight’s moves) subject to the rule that it cannot visit a square that it has already visited. Does the first or second player have a winning strategy under normal play?

(A knight’s move is two squares in one direction and one square in a perpendicular direction.)

4. Suppose that in a game of **Blue-Red Hackenbush**, Left (=Blue) tries to employ the greedy strategy of always making a move that maximizes the difference between the number of blue edges and the number of red edges. Give an example of a position in which Left will lose if she plays the greedy strategy but she can win playing nongreedily.