MA 524 Homework 10 – due 11/15

Homework:

1. Let $L$ be a distributive lattice, and fix any element $a \in L$. Let $\varphi: L \to L \times L$ be the map given by $\varphi(x) = (x \land a, x \lor a)$. Show that $\varphi$ is a lattice embedding, that is, $\varphi$ is an isomorphism onto its image and preserves meets and joins.

2. Let $L$ be a finite distributive lattice. Let $P$ be the subposet of join-irreducibles in $L$, and let $Q$ be the subposet of meet-irreducibles in $L$. Show that $P \cong Q$, and give an explicit isomorphism between them.

3. Let $P$ be a finite poset and $n$ any positive integer. Show that the number of order-preserving maps $\sigma: P \to n$ is a polynomial in $n$. What is its leading term?

4. Let $P$ be a finite poset. For any $t \in P$, let $\lambda_t = \# \{ s \mid s \leq t \}$. Show that

$$e(P) \geq \frac{|P|!}{\prod_{t \in P} \lambda_t}$$

and that equality holds if and only if $P$ does not contain the poset shown below as an induced subposet.

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