1. (a) For \( n \geq 1 \), prove that
\[
p_n - e_1 p_{n-1} + e_2 p_{n-2} - \cdots + (-1)^{n-1} e_{n-1} p_1 + (-1)^n n e_n = 0.
\]
What is the analogous equation involving \( p_n \) and \( h_n \)?
(b) Express \( h_n \) and \( e_n \) in terms of the basis \( \{ p_\lambda \} \).

2. Show that
\[
\sum_{\mu \vdash n} q^{\ell(\mu)} m_\mu = \sum_{j=0}^{n-1} (q - 1)^j s_{n-j,1^j}.
\]
Use this result to express \( p_n \) in terms of the basis \( \{ s_\lambda \} \).

3. (a) A Gelfand-Tsetlin pattern is a triangular array of nonnegative integers
\[
\begin{array}{cccc}
a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\
a_{22} & a_{23} & \cdots & & a_{2n} \\
a_{33} & & \cdots & & a_{3n} \\
\vdots & & \ddots & & \vdots \\
a_{nn} \\
\end{array}
\]
satisfying \( a_{ij} \leq a_{i+1,j+1} \leq a_{i,j+1} \) whenever these numbers are defined. (Each row is weakly increasing, and each number lies weakly between the two above it.)
Show that if \( \lambda \) is a partition of length \( n \), then the number of Gelfand-Tsetlin patterns with \( \lambda = (a_{1n}, a_{1,n-1}, \ldots, a_{12}, a_{11}) \) equals the number of semistandard Young tableaux of shape \( \lambda \) with largest entry at most \( n \).
(b) *Given a partition \( \lambda = (\lambda_1, \ldots, \lambda_n) \) and a positive integer \( k \), let \( k\lambda \) be the partition \((k\lambda_1, k\lambda_2, \cdots, k\lambda_n) \). Show that \( s_{k\lambda}(1,1,\ldots,1) \) is bounded above by a polynomial in \( k \) of degree at most \( \binom{n}{2} \).

4. Determine the image of the \( m \times n \) matrix consisting of all 1’s under the RSK correspondence.