Homework:

1. Prove the Erdős-Szekeres Theorem: any sequence of \( mn + 1 \) distinct real numbers either contains an increasing subsequence of length \( m + 1 \) or a decreasing subsequence of length \( n + 1 \).

2. Fix positive integers \( k \) and \( n \). For any partition \( \lambda \subseteq (n^k) \), define \( \lambda^\vee = (n - \lambda_k, n - \lambda_{k-1}, \ldots, n - \lambda_1) \). Show that \( c_\lambda^\mu = c_{\lambda^\vee}^{\mu^\vee} \). (Hence \( c_\lambda^\mu \) is symmetric in \( \mu, \nu \), and \( \lambda^\vee \).)

3. (a) Given a standard Young tableau \( T \) of size \( n \), let \( \Delta(T) \) be the tableau formed by removing the box labeled 1, rectifying the resulting tableau, and then decreasing all the numbers by 1. For instance, repeatedly applying \( \Delta \) to the tableau shown below gives:

\[
\begin{array}{ccc}
1 & 2 & 5 \\
3 & 6 \\
4 \\
\end{array}
\mapsto
\begin{array}{ccc}
1 & 4 \\
2 & 5 \\
3 \\
\end{array}
\mapsto
\begin{array}{ccc}
1 & 3 \\
2 & 4 \\
3 \\
\end{array}
\mapsto
\begin{array}{ccc}
1 & 2 \\
3 \\
2 \\
\end{array}
\mapsto
\begin{array}{ccc}
1 & 2 \\
1 \\
\end{array}
\mapsto
\emptyset
\]

The shapes of the tableaux \( T, \Delta(T), \Delta^2(T), \ldots \) define a saturated chain in Young’s lattice and hence a standard Young tableaux, which we call the evacuation of \( T \). For instance, in the example above,

\[
\text{evac}
\begin{pmatrix}
1 & 2 & 5 \\
3 & 6 \\
4 \\
\end{pmatrix}
= 
\begin{pmatrix}
1 & 3 & 6 \\
2 & 4 \\
5 \\
\end{pmatrix}
\].

Show that \( \text{evac(evac}(T)) = T \).

(It may be helpful to consider the growth diagram description of jeu de taquin. The map \( T \mapsto \text{evac}(T) \) is sometimes called the Schützenberger involution.)

(b) *Show that if the image of \( w = w_1 w_2 \cdots w_n \) under the RSK correspondence is \( (P, Q) \), then the image of the reverse \( w_n \cdots w_2 w_1 \) is \( (P^t, \text{evac}(Q)^t) \), where \( ^t \) denotes transpose.