1. Answer all questions in the spaces provided.
2. If you run out of room for an answer, continue on the back of the page.
3. Please show your work. You may not receive full credit if your work is not shown.
4. All answers should be in closed form.
5. No calculators, notes, or other outside assistance is allowed.
6. Each question is worth 10 points. The maximum score is 40.

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1. (10 points) How many 3-polytopes with at most 9 edges are there up to combinatorial equivalence? Draw a Schlegel diagram for each one, and indicate which ones are polar to one another.
2. (a) (5 points) Let $P \subseteq \mathbb{R}^d$ be a polytope and $L \subseteq \mathbb{R}^d$ an affine linear space. Show that if $L$ does not intersect $P$, then $L$ is contained in an affine hyperplane that does not intersect $P$.

(b) (5 points) Let $P$ be a polytope with vertex set $V$, and let $V' \subseteq V$. Show that $\text{conv}(V')$ is a face of $P$ if and only if the affine hull of $V''$ does not intersect the convex hull of $V\setminus V'$. 
3. (10 points) Let $P \subseteq \mathbb{R}^3$ be given by the following inequalities.

\[-x + 2y \leq 0\]
\[-x - y \leq 0\]
\[y + z \leq 1\]
\[2x - z \leq 4\]
\[3x - 4y - 2z \leq 6\]
\[x - 2y - z \leq 2\]

Find the maximum value of $x$ over all $(x, y, z) \in P$, and find the point(s) at which this maximum is attained.
4. Determine whether each of the following two diagrams is...

(a) (7 points) ...a regular subdivision.

(b) (3 points) ...combinatorially equivalent to a Schlegel diagram.