1. Suppose $P \subseteq \mathbb{R}^2$ is a lattice polygon with a “hole.” (In other words, $P = P_1 \setminus \text{int}(P_2)$ for some lattice polygons $P_1$ and $P_2$ with $P_2 \subseteq \text{int}(P_1)$.) Find a version of Pick’s theorem that holds for such polygons.

2. Show that for any positive integer $r \geq 1$, there exists a simplex $T_r$ in $\mathbb{R}^3$ with volume $\frac{r}{6}$ that contains no lattice points other than its vertices.

   (It follows that there is no direct analogue of Pick’s Theorem in dimension 3.)

3. Let $P$ be a finite poset. The order polytope $O(P) \subseteq \mathbb{R}^{\vert P \vert}$ is the set of all points $(x_s)_{s \in P}$ such that $0 \leq x_s \leq 1$ for all $s \in P$, and $x_s \leq x_t$ if $s \leq t$ in $P$. Show that the volume of $O(P)$ is $\frac{e(P)}{n!}$, where $e(P)$ is the number of linear extensions of $P$.

4. (a) Let $P \subseteq \mathbb{R}^2$ be a lattice polygon. Show that any point $(\frac{m}{2}, \frac{n}{2}) \in P$, where $m, n \in \mathbb{Z}$, is the midpoint of two lattice points in $P$.

   (b) Show that the analogous result does not hold in dimension 3.