MA 724 Homework 5 – due 2/14

1. Let $P$ be a polytope. How are the face posets of $P$ and $Q$ related if:
   
   (a) $Q$ is a pyramid with base $P$?
   
   (b) $Q$ is a bipyramid with base $P$?

   (c) $Q$ is a prism with base $P$?

2. Let $C \subseteq \mathbb{R}^d$ be a polyhedral cone, and define
   
   $C^\Delta = \{ c \in (\mathbb{R}^d)^* \mid cx \leq 0 \text{ for all } x \in C \}$.

   (a) Prove that
   
   $C^\Delta = \{ c \in (\mathbb{R}^d)^* \mid cx \leq 1 \text{ for all } x \in C \}$.

   (b) If $C = \text{cone}(Y) = P(A,0)$, give H- and V-representations of $C^\Delta$. Deduce that if $C$ is a pointed polyhedral cone of dimension $d$, then so is $C^\Delta$.

   (c) If $C$ is the homogenization of a polytope $P$, how does $C^\Delta$ relate to $P^\Delta$?

3. A polytope $P$ containing 0 in its interior whose vertices are lattice points is called reflexive if the vertices of $P^\Delta$ are lattice points in $(\mathbb{R}^d)^*$.

   (a) Show that if $P$ is reflexive, then 0 is the only lattice point in its interior.

   (b) Is the converse true?