1. Let $P \subseteq \mathbb{R}^d$ be a simple polytope, and let $c \in (\mathbb{R}^d)^*$ be a linear functional in general position with respect to $P$. Show that the number of vertices $v$ of $P$ that have exactly $k$ neighbors $w$ with $cw < cv$ is $h_k(P^\Delta)$.

(In particular, replacing $c$ with $-c$ proves the Dehn-Sommerville equations for $P^\Delta$.)

2. (a) Show that any $(\lfloor \frac{d}{2} \rfloor + 1)$-neighborly $d$-polytope is a simplex.

(Hint: You may find some previous homework problems helpful.)

(b) Show that any neighborly polytope of even dimension is simplicial.

(c) Show that there exist non-simplicial neighborly polytopes of any odd dimension $d \geq 3$.

3. Give a combinatorial condition for the subsets $I \subseteq [n]$ such that $\text{conv}(\{x(t_i) \mid i \in I\})$ is a face of the cyclic polytope $C_d(t_1, \ldots, t_n)$, where $x(t) = (t, t^2, \ldots, t^d)$ and $t_1 < t_2 < \cdots < t_n$.

(It follows that the combinatorial type of $C_d(t_1, \ldots, t_n)$ does not depend on the $t_i$.)