1. (a) Let \( n \geq 2 \) be an integer. Given \( 2n - 2 \) generic \((n - 2)\)-dimensional subspaces of \( \mathbb{P}^n \), how many lines intersect all of them?

(b) Show that, as a projective variety under the Plücker embedding, the degree of \( G(k, n) \) is the number of \textit{standard Young tableau} of shape \( k \times (n - k) \), that is, the number of ways to place \( 1, 2, \ldots, k(n - k) \) in a \( k \times (n - k) \) rectangle such that each row and column is increasing.

2. *Let \( \geq \) denote dominance order on partitions of \( n \).

(a) *Let \( \lambda, \mu \vdash n \). Show that \( \lambda \geq \mu \) if and only if \( \mu' \geq \lambda' \).
(Here, \( \lambda' \) is the conjugate partition of \( \lambda \).)

(b) *Describe the covering relations in dominance order.

(c) *Let \( \lambda \) and \( \mu \) be partitions of \( n \) with (at most) \( k \) parts. Show that \( \lambda \geq \mu \) if and only if \( \mu \in \mathbb{R}^k \) lies in the convex hull of the points (in \( \mathbb{R}^k \)) whose coordinates are permutations of \( \lambda \).

3. The \textit{power sum symmetric functions} \( p_\lambda \) are defined by \( p_\lambda = p_{\lambda_1}p_{\lambda_2} \cdots \), where \( p_n = \sum_i x_i^n \). Show that \( \{p_\lambda \mid \lambda \vdash n\} \) is a basis for \( \Lambda^n_{\mathbb{Q}} \).

Do the \( p_\lambda \) form a \( \mathbb{Z} \)-basis of \( \Lambda^n_{\mathbb{Z}} \)?

4. Given a basis \( \{b_\lambda\} \) of \( \Lambda \), we say a symmetric function \( f \) is \( b \)-positive if the coefficients of \( f \) when expanded in the basis \( \{b_\lambda\} \) are nonnegative.

Describe all symmetric functions that are both \( e \)-positive and \( h \)-positive.