Please discuss your final project topics with me by 4/3.

1. Show that the number of Yamanouchi words of length $n$ equals the number of involutions $w \in S_n$ (i.e., $w^2 = id$).

2. A hook shape $\nu$ is a partition that does not contain a $2 \times 2$ square, that is, $\nu = (a, 1^b)$ for some $a, b \geq 0$. Show that the skew shape $\lambda/\mu$ contains a $2 \times 2$ square if and only if $c^\lambda_{\mu\nu} = 0$ for all hook shapes $\nu$.

3. (a) Given a standard Young tableau $T$ of size $n$, let $\Delta(T)$ be the tableau formed by removing the box labeled 1, rectifying the resulting tableau, and then decreasing all the numbers by 1. For instance, repeatedly applying $\Delta$ to the tableau shown below gives:

\[
\begin{array}{ccc}
1 & 2 & 5 \\
3 & 6 \\
4 \\
\end{array}
\mapsto
\begin{array}{ccc}
1 & 4 \\
2 & 5 \\
3 \\
\end{array}
\mapsto
\begin{array}{ccc}
1 & 3 \\
2 & 4 \\
3 \\
\end{array}
\mapsto
\begin{array}{ccc}
1 & 2 \\
3 \\
2 \\
1 \\
\mapsto
\begin{array}{ccc}
1 \\
2 \\
1 \\
\emptyset
\end{array}
\end{array}
\]

The shapes of the tableaux $T$, $\Delta(T)$, $\Delta^2(T)$, \ldots define a chain of partitions and hence a standard Young tableau, which we call the evacuation of $T$. For instance, in the example above,

$$\text{evac}\left(\begin{array}{ccc}
1 & 2 & 5 \\
3 & 6 \\
4 \\
\end{array}\right) = \begin{array}{ccc}
1 & 3 & 6 \\
2 & 4 \\
5 \\
\end{array}.$$

Using growth diagrams, show that $\text{evac}(\text{evac}(T)) = T$.
(The map $T \mapsto \text{evac}(T)$ is sometimes called the Schützenberger involution.)

(b) Show that if the image of $w = w_1w_2\cdots w_n$ under the RSK correspondence is $(P, Q)$, then the image of the reverse $w_n \cdots w_2w_1$ is $(P^t, \text{evac}(Q)^t)$, where $^t$ denotes transpose.