1. For a permutation \( w \in S_n \), let \( K(w) \) be the tableau of staircase shape whose \( i \)th column contains the numbers \( w_1, \ldots, w_{n-i} \), rearranged to be in increasing order. For instance,

\[
K(326154) = \begin{array}{cccc}
1 & 1 & 2 & 2 \\
2 & 2 & 3 & 3 \\
3 & 3 & 6 & 6 \\
5 & 6 & 6 & 6
\end{array}
\]

Show that \( v \preceq w \) in Bruhat order if and only if \( K(v) \preceq K(w) \) entrywise.

2. The Lehmer code of a permutation \( w \in S_n \) is the sequence \( c(w) = (c_1, c_2, \ldots, c_n) \), where \( c_i = \# \{ j > i \mid w_j < w_i \} \).

Show that the smallest term in \( \mathcal{S}_w \) in lexicographic order is \( x^{c(w)} \). (Here \( x^\alpha < x^\beta \) in lexicographic order if, at the smallest index \( i \) where \( \alpha \) and \( \beta \) differ, \( \alpha_i < \beta_i \).

3. Let \( v \in S_n \), and let \( w \in S_\infty \) such that \( w_i = i \) for \( 1 \leq i \leq n \). Show that \( \mathcal{S}_{vw} = \mathcal{S}_v \mathcal{S}_w \).

4. (a) Given constants \( c_1, \ldots, c_n \), find the expansion of

\[
(c_1 x_1 + \cdots + c_n x_n) \mathcal{S}_w
\]

in the Schubert basis.

(b) Let \( w \in S_\infty \) have largest descent \( p \), and let \( q \) be the largest integer for which \( w(q) < w(p) \). Show that

\[
\mathcal{S}_w = x_p \mathcal{S}_{wtpq} + \sum_{v \in T} \mathcal{S}_v
\]

for some set of permutations \( T \).

(Such an expression, due to Lascoux and Schützenberger, is called a transition.)